Paper Reference(s)

6681

Edexcel GCE

Mechanics M5

Advanced Level

Tuesday 23 June 2009 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Orange or Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M4), the paper reference (6680), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 6 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

M34277A

1. At time $t = 0$, a particle P of mass 3 kg is at rest at the point A with position vector $(\mathbf{j} - 3\mathbf{k})$ m. Two constant forces $\mathbf{F}_1$ and $\mathbf{F}_2$ then act on the particle P and it passes through the point B with position vector $(8\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$ m.

Given that $\mathbf{F}_1 = (4\mathbf{i} - 2\mathbf{j} + 5\mathbf{k})$ N and $\mathbf{F}_2 = (8\mathbf{i} - 4\mathbf{j} + 7\mathbf{k})$ N and that $\mathbf{F}_1$ and $\mathbf{F}_2$ are the *only* two forces acting on P, find the velocity of P as it passes through B, giving your answer as a vector.

(7)

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2. At time t seconds, the position vector of a particle P is $\mathbf{r}$ metres, where $\mathbf{r}$ satisfies the vector differential equation

$$\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} = 4\mathbf{r} = \mathrm{e}^{2t}\mathbf{j}.$$

When $t = 0$, P has position vector $(\mathbf{i} + \mathbf{j})$ m and velocity $2\mathbf{i}$ m s^{-1}.

Find an expression for $\mathbf{r}$ in terms of t .

(11)

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3. A spaceship is moving in a straight line in deep space and needs to increase its speed. This is done by ejecting fuel backwards from the spaceship at a constant speed c relative to the spaceship. When the speed of the spaceship is v, its mass is m.

(*a*) Show that, while the spaceship is ejecting fuel,

$$\frac{\mathrm{d}v}{\mathrm{d}m} = -\frac{c}{m}.$$

(5)

The initial mass of the spaceship is m_0 and at time t the mass of the spaceship is given by $m = m_0(1 - kt)$, where k is a positive constant.

(*b*) Find the acceleration of the spaceship at time t .

(4)

4.

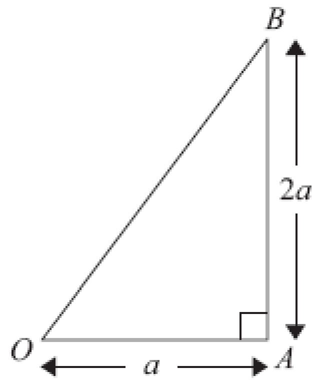

Figure 1

A uniform lamina of mass M is in the shape of a right-angled triangle OAB. The angle OAB is 90°, $OA = a$ and $AB = 2a$, as shown in Figure 1.

(*a*) Prove, using integration, that the moment of inertia of the lamina OAB about the edge OA is $\frac{2}{3}Ma^2$.

(You may assume without proof that the moment of inertia of a uniform rod of mass m and length $2l$ about an axis through one end and perpendicular to the rod is $\frac{4}{3}ml^2$.)

(6)

The lamina OAB is free to rotate about a fixed smooth horizontal axis along the edge OA and hangs at rest with B vertically below A. The lamina is then given a horizontal impulse of magnitude J. The impulse is applied to the lamina at the point B, in a direction which is perpendicular to the plane of the lamina. Given that the lamina first comes to instantaneous rest after rotating through an angle of 120°,

(*b*) find an expression for J, in terms of M, a and g .

(7)

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5. Two forces $\mathbf{F}_1 = (2\mathbf{i} + \mathbf{j})$ N and $\mathbf{F}_2 = (-2\mathbf{j} - \mathbf{k})$ N act on a rigid body. The force $\mathbf{F}_1$ acts at the point with position vector $\mathbf{r}_1 = (3\mathbf{i} + \mathbf{j} + \mathbf{k})$ m and the force $\mathbf{F}_2$ acts at the point with position vector $\mathbf{r}_2 = (\mathbf{i} - 2\mathbf{j})$ m. A third force $\mathbf{F}_3$ acts on the body such that $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ are in equilibrium.

(*a*) Find the magnitude of $\mathbf{F}_3$.

(4)

(*b*) Find a vector equation of the line of action of $\mathbf{F}_3$.

(8)

The force $\mathbf{F}_3$ is replaced by a fourth force $\mathbf{F}_4$, acting through the origin O, such that $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_4$ are equivalent to a couple.

(*c*) Find the magnitude of this couple.

(4)

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6. A pendulum consists of a uniform rod AB, of length $4a$ and mass $2m$, whose end A is rigidly attached to the centre O of a uniform square lamina $PQRS$, of mass $4m$ and side a. The rod AB is perpendicular to the plane of the lamina. The pendulum is free to rotate about a fixed smooth horizontal axis L which passes through B. The axis L is perpendicular to AB and parallel to the edge PQ of the square.

(*a*) Show that the moment of inertia of the pendulum about L is $75ma^2$.

(4)

The pendulum is released from rest when BA makes an angle α with the downward vertical through B, where $\tan \alpha = \frac{7}{24}$. When BA makes an angle θ with the downward vertical through B, the magnitude of the component, in the direction AB, of the force exerted by the axis L on the pendulum is X.

(*b*) Find an expression for X in terms of m, g and θ.

(9)

Using the approximation $\theta \approx \sin \theta$,

(*c*) find an estimate of the time for the pendulum to rotate through an angle α from its initial rest position.

(6)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.	$\pm(8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k})$	B1
	$((4\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}) + (8\mathbf{i} - 4\mathbf{j} + 7\mathbf{k})).(8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k}) = \frac{1}{2}3v^2$	M1 A1 f.t.
	$12 = v$	A1
	$\mathbf{v} = \dfrac{12}{\sqrt{8^2 + (-4)^2 + 8^2}}(8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k})$	M1
	$\mathbf{v} = (8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k})\,\text{ms}^{-1}$	M1 A1
		(7 marks)
2.	C.F. is $\mathbf{r} = \mathbf{A}\cos 2t + \mathbf{B}\sin 2t$	B1
	P.I. is $\mathbf{r} = \mathbf{p}e^{2t}$	B1
	$\dot{\mathbf{r}} = 2\mathbf{p}e^{2t}$ $\ddot{\mathbf{r}} = 4\mathbf{p}e^{2t}$	B1 ft
	$4\mathbf{p}e^{2t} + 4\mathbf{p}e^{2t} = \mathbf{j}e^{2t}$	M1
	so, (PI is) $\mathbf{r} = \frac{1}{8}\mathbf{j}e^{2t}$	A1
	GS is $\mathbf{r} = \mathbf{A}\cos 2t + \mathbf{B}\sin 2t + \frac{1}{8}\mathbf{j}e^{2t}$	A1 ft
	$t = 0,\ \mathbf{r} = \mathbf{i} + \mathbf{j} \Rightarrow\ \mathbf{i} + \mathbf{j} = \mathbf{A} + \frac{1}{8}\mathbf{j} \Rightarrow\ \mathbf{i} + \frac{7}{8}\mathbf{j} = \mathbf{A}$	M1 A1
	$\dot{\mathbf{r}} = -2\mathbf{A}\sin 2t + 2\mathbf{B}\cos 2t + \frac{1}{4}\mathbf{j}e^{2t}$	M1 A1
	$t = 0,\ \dot{\mathbf{r}} = 2\mathbf{i} \Rightarrow\ 2\mathbf{i} = 2\mathbf{B} + \frac{1}{4}\mathbf{j} \Rightarrow\ \mathbf{i} - \frac{1}{8}\mathbf{j} = \mathbf{B}$	
	$\mathbf{r} = (\mathbf{i} + \frac{7}{8}\mathbf{j})\cos 2t + (\mathbf{i} - \frac{1}{8}\mathbf{j})\sin 2t + \frac{1}{8}\mathbf{j}e^{2t}$	A1
		(11 marks)

Question Number	Scheme	Marks
3. (a)	$mv = (m+\delta m)(v+\delta v) - (-\delta m)(c-v)$	M1 A2
	$mv = mv + m\delta v + v\delta m + c\delta m - v\delta m$	
	$-m\delta v = c\delta m$	
	$\frac{\mathrm{d}v}{\mathrm{d}m} = -\frac{c}{m}$ *	M1 A1 (5)
(b)	$\frac{\mathrm{d}m}{\mathrm{d}t} = -m_0 k$	B1
	$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}v}{\mathrm{d}m} \text{ x } \frac{\mathrm{d}m}{\mathrm{d}t}$	M1
	$= -\frac{c}{m} \text{ x } -m_0 k$	
	$= \frac{cm_0 k}{m_0(1-kt)}$	M1
	$= \frac{ck}{(1-kt)}$	A1 (4)
		(9 marks)

Question Number	Scheme	Marks
4. (a)	$\delta m = \dfrac{2Mx\delta x}{a^2}$	M1 A1
	$\delta I = \dfrac{1}{3}\dfrac{2Mx\delta x}{a^2}(2x)^2$	M1 A1
	$I = \int_0^a \dfrac{8Mx^3\mathrm{d}x}{3a^2}$	
	$= \dfrac{8M}{3a^2}\left[\dfrac{x^4}{4}\right]_0^a$	M1
	$= \dfrac{2}{3}Ma^2$ *	A1 (6)
(b)	$J.2a = \dfrac{2}{3}Ma^2\omega$	M1 A1
	$\dfrac{1}{2}\dfrac{2}{3}Ma^2\omega^2 = Mg\dfrac{2a}{3}(1+\cos 60^o)$	M1 A2
		M1
	solving for J	
	$J = M\sqrt{\dfrac{ag}{3}}$	A1 (7)
		(13 marks)

Question Number	Scheme	Marks
5. (a)	$(2\mathbf{i} + \mathbf{j}) + (-2\mathbf{j} - \mathbf{k}) + \mathbf{F}_3 = \mathbf{0}$	M1
	$\mathbf{F}_3 = -2\mathbf{i} + \mathbf{j} + \mathbf{k}$	A1
	$\lvert\mathbf{F}_3\rvert = \sqrt{(-2)^2 + 1^2 + 1^2} = \sqrt{6}$ N	M1 A1 (4)
(b)	$(3\mathbf{i} + \mathbf{j} + \mathbf{k})$ x $(2\mathbf{i} + \mathbf{j})$ + $(\mathbf{i} - 2\mathbf{j})$ x $(-2\mathbf{j} - \mathbf{k})$ + $(x\mathbf{i} + y\mathbf{j} + z\mathbf{k})$ x $(-2\mathbf{i} + \mathbf{j} + \mathbf{k})$	M1
	$(-\mathbf{i} + 2\mathbf{j} + \mathbf{k}) + (2\mathbf{i} + \mathbf{j} - 2\mathbf{k}) + ((y - z)\mathbf{i} + (-2z - x)\mathbf{j} + (x + 2y)\mathbf{k})$	A3
	$y - z = -1$, $-x - 2z = -3$, $x + 2y = 1$	M1
	$x = 1$, $y = 0$, $z = 1$ is a solution	M1
	so, $\mathbf{r} = (\mathbf{i} + \mathbf{k}) + \lambda(-2\mathbf{i} + \mathbf{j} + \mathbf{k})$ is a vector equn of line of action of $\mathbf{F}_3$	M1 A1 (8)
(c)	$(3\mathbf{i} + \mathbf{j} + \mathbf{k})$ x $(2\mathbf{i} + \mathbf{j})$ + $(\mathbf{i} - 2\mathbf{j})$ x $(-2\mathbf{j} - \mathbf{k})$ = $\mathbf{G}$	M1
	$(-\mathbf{i} + 2\mathbf{j} + \mathbf{k}) + (2\mathbf{i} + \mathbf{j} - 2\mathbf{k}) = (\mathbf{i} + 3\mathbf{j} - \mathbf{k}) = \mathbf{G}$	A1
	$\lvert\mathbf{G}\rvert = \sqrt{1^2 + 3^2 + (-1)^2} = \sqrt{11}$ N m	M1 A1 (4)
		(16 marks)

Question Number	Scheme	Marks
6. (a)	$\frac{1}{3}2m(4a)^2 + \frac{1}{12}4ma^2 + 4m(4a)^2$	B1 M1 A1
	$= \frac{32}{3}ma^2 + \frac{1}{3}ma^2 + 64ma^2$	
	$= 75ma^2$ *	A1 (4)
(b)	$\frac{1}{2}75ma^2\omega^2 = 2mg2a(\cos\theta - \cos\alpha) + 4mg4a(\cos\theta - \cos\alpha)$	M1 A2
	$a\omega^2 = \frac{8}{15}g(\cos\theta - \frac{24}{25}) = \frac{8}{375}g(25\cos\theta - 24)$	A1
	$X - 6mg\cos\theta = 2m2a\omega^2 + 4m4a\omega^2 = 20ma\omega^2$	M1 A2
	$X = 6mg\cos\theta + 20m\frac{8}{375}g(25\cos\theta - 24)$	M1
	$= \frac{50mg\cos\theta}{3} - \frac{256mg}{25}$	A1 (9)
(c)	$-2mg2a\sin\theta - 4mg4a\sin\theta = 75ma^2\ddot{\theta}$	M1 A1
	$\ddot{\theta} = -\frac{4g}{15a}\sin\theta$	A1
	$\approx -\frac{4g}{15a}\theta$, SHM	M1
	Time = $\frac{1}{4}2\pi\sqrt{\frac{15a}{4g}}$	M1
	$= \frac{\pi}{4}\sqrt{\frac{15a}{g}}$	A1 (6)
		(19 marks)